Numerical Analyses of Piled Raft Foundation in Soft Soil Using 3D-FEM

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ABSTRACT: In recent years, the piled raft foundation has been widely accepted as one of the most economical methods of foundation systems. To evaluate the possibility of implementing this system in a very soft ground condition, this research performed the numerical analyses of the piled raft and pile group foundation systems for low-rise (8-storey) and high-rise (25-storey) buildings with 1-2 basement levels in subsoil conditions of the central part of Thailand, using three-dimensional Finite Element Method. The soils are modelled with Hardening Soil model and Mohr-Coulomb model. Evaluations of the performances of piled raft foundation, i.e., the load sharing ratio of piles, settlement behaviours in both the foundation system and the raft are discussed in comparison with those of the pile group. With limited cases considered and assumptions in this study, the results suggest the potential of using the piled raft system for low-rise building having 2 levels of basement. With this condition, the raft can carry some bearing capacity from pile around 20%. However, the safety factor of piles in the foundation design seems to play a key role on the effectiveness of the piled raft foundation as well and should be further studied.

Keywords: Piled rafts, 3D FEM, Soft soil, Piled-raft load sharing.

1. INTRODUCTION

The number of tall building construction in central part of Thailand has been continuously increasing during these two decades. In some urban areas the tall buildings cannot be constructed. According to the law, the high rise building is not allowed to construct in the area of which the fire-fighting vehicle cannot access. Therefore, the low-rise building have becomes more popular in recent years. With the current law, the height of the building is limited for approximate 8-storey building. Typically, new office or residential buildings require 1 or 2 basements for utilizing as a car park space. As the subsoil of this area is soft clay interspersed with sand, the pile foundation must be used to transfer the load to the stiff soil layers. Generally, for this kind of building, the mat foundation (raft) has been chosen for the intermediary in the transfer load of buildings onto piles.

In Thailand, the designers prefer to consider the pile group to support a structure (Amornfa et al., 2012). The pile groups mostly focus on pile capacity and group settlement without considering the presence of the raft or mat. However, in fact the foundations are built using concrete and their bottom surfaces are attached to the soil beneath. Therefore, in most cases ends up with overdesign of the foundation. In recent years, the foundation engineers tend to combine these two separate systems (between shallow foundations (rafts) and deep foundations (piles)). Such a foundation system is referred to as piled raft foundation.

Nowadays, the "piled raft foundation" has been widely used for many structures, particularly high rise buildings. Piled rafts have proved to be an economical alternative compared to the conventional pile foundations in conditions in which the soil below the raft can provide significant bearing capacity (Randolph, 1994, Poulos, 2001). Thus, the piled raft systems have been used extensively in many parts of the world (Poulos & Davis, 1980, Randolph, 1983, Yamashita et al., 1994, Kachzenbach et al., 2000). The application of piled rafts on soft ground is becoming a significant issue in foundation design. Normally, the design and construction of foundation system on soft ground have posed various problems to geotechnical engineers, such as excessive settlement, negative skin friction and bearing capacity failure.

Despite these concerns, a few successful applications of piled rafts on soft ground have been reported (Yamashita et al., 1998, Poulos, 2005, Tan et al., 2006). A piled raft includes three elements of pile, raft and subsoil. The behaviour of a piled raft is thus affected by the complex interaction among the piles, subsoil and raft. In order to solve this complex problem, several methods for the analysis of the piled raft foundations have been developed. Poulos and Davis (1980), and Randolph (1983) carried out the early work on simplified calculation methods. With an advancement of the computer, more rigorous methods such as Finite Element Method (FEM) are also used in some of recent researches (Reul, 2004, Jaeyeon et al., 2012).

In this paper, three dimensional (3D) finite elements (FE) method using PLAXIS 3D program, is used to analyse the behaviour of piled raft in soft ground. Two different building sizes, i.e., low-rise (8storey) and high-rise (25-storey) buildings with basements are considered in this study to evaluate the potential of using the piled raft system. The main factor to be investigated its influence is the level of raft.

2. PILED RAFT FOUNDATION CONCEPT

The Piled Raft Foundation (PRF) is a composite construction that combines the bearing effect of both foundation elements (piles and raft). As shown in Figure 1. Both, piles and raft are considered in the load distribution process:

$$P_{tot} = P_p + P_r \tag{1}$$

where P_{tot} = total load of the building; P_p = load carried by the pile group; P_r = load carried by the raft.

The PRF allows the reduction of total settlements and differential settlements in a very economical way compared to α traditional foundation concepts due to the contribution of both the piles and the raft (Katzenbach et al., 2000).

The bearing behaviour of the piled raft is commonly described by the piled raft coefficient or the load sharing ratio of piles which is defined by the ratio between the sum of load carried by pile and the total load of the building:

$$\alpha_{pr} = \frac{\sum R_{pile,i}}{R_{tot}} \tag{2}$$

where α_{pr} = the load sharing ratio of piles; $\sum R_{pile,i}$ = the amount of the pile loads; R_{tor} = total load of the structure.



Figure 1 Concept of piled raft foundation (after Katezenbach et al. 2000)

3. REFERENCE CASE AND PROBLEM CHARACTERISTICS

3.1 Reference case

A 9 m \times 9 m \times *t* square raft with 9 piles is considered in this study, where t is thickness of raft. The study considers a low-rise (8-storey) and high-rise (25-storey) buildings with basements having rigid raft (0.5 and 1 m thick) as considered from Horikoshi and Randolph (1997). Both piled raft and pile group having identical characteristics are considered as shown in Figures 2 (a) and 2 (b), respectively.

For the piled raft foundation concept (a), the raft is actually directly contacted to the soil beneath. Some portion of the load is taken by raft through soil. On the other hand, for piled group foundation concept (b), all building loads are carried by piles without raft contribution. In 3D FEM simulation, the raft must then set to have sufficient clearance to the ground beneath to assure the raft would not touch the soil.



Figure 2 Basic problems analysed (a) piled raft, (b) pile group

The piles in the foundations for case having raft level at the ground surface are designed to carry the allowable load with a safety factor (FS) of 2.3. The bored piles have diameter (d) of 1 m being arranged in the foundation with spacing of 3d and the level of pile tip is at 23 (1st stiff clay layer) and 36 m (2nd sand layer) below the ground surface for low-rise and high-rise buildings, respectively. The raft level is varied from 0 to 10 m below the ground level. With fixed pile tips, this means that the FS of piles decreases with deeper raft level due to the decreasing length of piles. A summary of the analysis cases is shown in Table 1.

Table 1 Summary of piled raft foundation of numerical analyses conducted

Building	Pile spacing	Pile tip level (m)	Raft thickness (m)	Raft level (m)		
Low-rise	3 <i>d</i> *	23 ^{f**}	0.5, 1	0,4,8,10		
High-rise	3 <i>d</i> *	36 ^{e**}	0.5, 1	0,4,8,10		

* d (pile diameter): 1 m.

** f: floating pile in clay; e: end bearing in sand layer.

3.2 Subsoil condition

The subsoil conditions in this study are referred to those in the north of Bangkok. The generalized profiles of the stratified soil at the considered location are shown in Figure 3. The uppermost 2.0 m thick layer is the weathered crust, which is underlain by 6.0 m thick soft to medium clay layer. A medium clay layer is found at the depth of 8.0 m from the surface. Below the medium clay is stiff clay; the thickness is about 15m. The first sand layer is generally found at a depth of 25 to 30m. Below the upper first sand layer, there is stiff clay and further down alternating layers of dense sand and hard clay the ground water table is below the ground surface at 1.5 m.

3.3 Applied load

It is customary to use Uniformly Distributed Loads (UDL) in the analysis of piled raft. From the computation on the weight of the structure and designed load, the UDL of 140 kPa is considered for the low-rise case, hereafter designated as UDL140. For high-rise case, the UDL of 350 kPa is used to apply, hereafter designated as UDL350.



Figure 3 Soil profile in this study (Jamsawang et al., 2010)

The basement is considered to apply the load of 50 ton per level. It is noted that the self-weight of the raft and piles are applied through gravitational force and therefore not been included in the UDL. The total applied loads on each foundation are listed in Table 2. These UDL are applied on top surface of the raft in analyses of piled raft, pile group and raft.

Table 2 Summary of total applied load on foundation in numerical analyses conducted

Building	Raft level (m)	Total load (kPa)
Low-rise	0	140
(8-storey)	4	146
	8	152
	10	158
High-rise	0	350
(25-storey)	4	356
	8	362
	10	368

4. FINITE ELEMENT MODELING OF FOUNDATIONS

4.1 Finite element mesh and boundary condition

The 3D FEM using PLAXIS 3D was carried out in this study. The 3D model included a rigorous treatment of the soil and raft which were represented by volume elements. The piles are modeled as embedded piles in which the pile is assumed to be a slender beam element. Figure 4 shows a typical 3D FE mesh used in this analysis. The boundary conditions adopted for analyses are displacement restraints with roller supports applied on all vertical sides and pin supports applied to the base of the mesh.

4.2 Soil constitutive models and parameters

The soft clay, medium clay and first stiff clay layers were modelled with a Hardening Soil Model with small strain using the soil parameters from Detkhong and Jongpradist (2014). The 1st -2nd sand, 2nd stiff clay and hard clay layer were modelled with a Mohr–Coulomb model. The soil properties used in the analyses are mainly determined from correlating local investigated data with comprehensive in situ tests of MRT projects (Prust et al., 2005) and previous laboratory tests from Asian Institute of Technology (AIT). Table 3 summarizes the material parameters used in the analyses. All the analyses are based on the undrained condition. The accuracy of simulations for geotechnical work in Bangkok subsoil by the selected models with the calibrated material parameters has been validated with measured data of well-documented case histories of tunnel excavations (Detkhong and Jongpradist, 2014).

4.3 Post analysis

The average settlement between that at the pile head and that at the raft is commonly used to represent the settlement of the foundation system in previous works. In this study, the new method was developed to represent the average settlement, S_{avg} . It is defined as the ratio between the settlement volume, $V_{settlement}$ and area of foundation, A_f as shown in Equation (3).

$$s_{avg} = \frac{V_{settlement}}{A_f}$$
(3)

The $V_{settlement}$ is the volume under settlement profile over the area of the foundation as illustrated in Figure 5. This method provides more precise value of the average settlement.



Figure 4 Geometry of the problem and 3D Finite element mesh used in this study

Material		Model	$\begin{array}{c} \gamma_t \\ (kN/m^3) \end{array}$	Material behaviour	S _u (kPa)	C' (kPa)	ø ()	E _{u,} E' (kPa)	$\mathbf{E}_{50}^{\mathrm{ref}}, \mathbf{E}_{\mathrm{oed}}^{\mathrm{ref}}$	E ^{ref} ur (kPa)	G ^{ref} ₀ (kPa)	γ _{0.7}	m	p _{ref} (kPa)	v, v _{ur}
Subsoil	Depth (m.)														
Weathered clay	0-2	MCM	17	Undrained	40			6000							0.3
Soft clay	2-8	HSS	15.2	Undrained		0	23		7000	23280	8954	1x10 ⁻⁴	1	100	0.33
Medium clay	8-10	HSS	18.4	Undrained		0	24		10300	30900	22800	1x10 ⁻⁴	1	100	0.32
1st Stiff clay	10-25	HSS	19	Undrained		0	26		25400	83900	32270	2x10 ⁻³	1	552	0.32
1st Sand	25-28	MCM	20	Drained		-	36	85800							0.3
2nd Stiff clay	28-35	MCM	20	Undrained	192			96000							0.3
2nd Sand	35-46	MCM	20	Drained		-	37	96200							0.3
Hard clay	46-60	MCM	20	Undrained	223			111500							0.3
Foundation															
Bored Pile	Tip -23,-36	LEM	6-8	Non-porous				2.6×10^7							0.2
Raft	0,-4,-8,-10	LEM	24	Non-porous				2.8×10^{7}							0.2

Table 3 Constitutive models and model parameters used in analyses

HSS: Hardening Soil Model with small strain; MCM: Mohr-Coulomb model; LEM: Linear Elastic Model

The differential settlement (Δs) was calculated by the maximum settlement of raft (S_{max}) and the minimum settlement of raft (S_{min}) as shown in Equation (4).





Figure 5 The settlement profile over the area of foundation

5. COMPUTED RESULTS

5.1 Effect of raft level on the load sharing ratio of piles

Figure 6 shows the load sharing ratio of piles for different raft levels below the ground surface and raft thicknesses for both building types. The analysis results show that when the raft was placed on the medium clay layer, the load sharing ratio of pile has been decreased significantly. For subsoil condition and problem characteristics in this study, the load sharing ratio reduces to 81% and 90% for the low-rise and high-rise buildings respectively. When the raft level is placed on the stiff clay layer, the load shared by piles becomes 75% and 86%. This raft level is considered to be equivalent to two levels of basement. This indicates the potential of using piled raft system for low-rise building having underground basement in soft soil condition.

However, for building of which the raft level is still in the soft clay layer, the load sharing ratios of piles are larger than 86% and 92.5% of the low rise and high rise building loads, respectively. Thus the piled raft foundation design concept does not offer much benefit for this condition. Therefore, the piled raft system for building in soft ground will be feasible for two levels of basement. In the figure, no significant influence of raft thickness (0.5 and 1 m) on load sharing ratio can be seen.



Figure 6 Load sharing ratio of piles (α_{pr}) versus raft level of different building types and raft thickness

5.2 Effect of raft level on settlement of PRF

The average settlements of Piled Raft Foundation (PRF) for different raft levels and raft thicknesses are illustrated in Figure 7. For low-rise building, the analysis results show that the average settlement increases with increasing raft level. This is because the piles in the case of deeper raft level are shorter (the FS is also smaller) and then their settlements become larger with the load applied. No significant difference on average settlement between cases with 0.5 and 1 m thick rafts with the same raft level can be seen. However, the differential settlement of the PRF ($s_{max} - s_{min}$ in Eq. (4)) with 0.5 m thick raft is much larger than that of 1.0 m thick raft and continues increasing with raft level (Figure 8). Whereas, the differential settlements of the PRF with 1.0 m thick raft keep constant with increasing raft level. This implies that the thickness of raft in the PRF is a key parameter to reduce differential settlement.

For the case of high-rise building, with increasing raft level, the average settlements of the PRF become smaller. Unlike the low-rise case, difference of the average settlement between PRFs having different raft thickness can be seen. In Figure 8, the differential settlement does not vary much with level of the raft. However, the differential settlements of PRF with 0.5 m thick raft are much greater than those of PRF with 1.0 m thick raft.



Figure 7 Average settlement of PRF versus raft level of different building types and raft thickness



Figure 8 Differential settlement in the raft of PRF versus raft level of different building types and raft thickness

5.3 The load-settlement curves of pile and foundation

Figure 9 illustrated the computed load-settlement curves of the PRF, pile group (PG), raft in PRF, piles in PRF and raft alone of low rise and high rise building cases. Only the case with raft level on the medium clay (-8 m) is considered in this part. In the figure, at the design load (152 kPa) of low rise building (Figure 9(a)) the settlement of PRF equals 10.64 mm. With the same settlement, the PG can carry less load than PRF around 19% or 28.88 kPa. At the same carrying load (152 kPa), the settlement of the PRF is less than that of the PG. This means that the raft in PRF enhances not only the load capacity but also the deformation of the PRF system. The load-settlement curves of the piles in PRF and PG are identical for the range of load less than the designed load. Beyond this, slight difference can be seen. For the same settlement, the deformation of raft in PRF is less than that of the raft (alone).

Figure 9(b) shows the analysis results of high rise building case. At the design load (362 kPa), the PRF settles 12.82 mm. Considering the same magnitude of settlement, the PG can carry less load than PG around 10.3% or 37.3 kPa. Conclusion, the PRF can receive more load than PG for both low rise and high rise building when PRF concept is applied.



Figure 9 Load-settlement curves of piled raft(PRF), piled group, raft in piled raft and raft alone with II basements. (a) Low rise (LR); (b) High rise (HR); (c) Low rise and high rise buildings

Similar to the previous condition, the load-settlement curves of the piles in PRF and PG are identical for the range of load less than the designed load. This confirms that the behaviour of piles in PRF is as same as that in the PG.

In Figure 9(c), comparison of the analysis results between low rise and high rise building cases is made. It can be seen that at the design load the settlements of PRF and PG for low rise building are higher than those of the high rise building. Moreover, at the design load, the difference of settlements between PRF and PG in case of low rise building is much larger than that of between PRF and PG in case of high rise building. This indicates that some mobilized deformation must be allowed to enhance the efficiency of the PRF system; this is thought to be due to the use of floating pile in low rise building. It is noticed that the load settlement curves of raft in PRF for both the high rise and low rise buildings are almost the same. This is due to the similarity of soil profile and foundation geometry.

The behaviour of piles in the PRF is hereafter investigated. Figure 10 illustrates the computed load-settlement curves of the edge piles in the PRF of low rise and high rise buildings with various raft levels. The analysis results show that, for low rise buildings, the loadsettlement curves gradually shift to the left side with increasing raft level. This implies that the capacity of piles in PRF decreases with increasing raft level. For high rise buildings, the difference can be seen only after increasing the raft level from 0 to 4 m. With further increasing the raft levels to 8 and 10 m, no significant difference can be seen. The capacity of piles in low rise buildings decrease with increasing raft levels rather than that in the high rise building. It is noted that, for low rise buildings, the original length of the piles is 23 m. With increasing the raft level up to 10 m would result to shortening of pile of larger than 40%. The FS of shorter piles might significantly decreases. Whereas, for high rise building of which the original length of pile is 36 m, the shortening of pile is 27%. With stiffer soil in deeper stratum at which the pile tip is embedded, not much change in FS is expected.



Figure 10 Load-settlement curves of edge piles in PRF for various raft levels

Figure 11 shows the comparison of load ratio shared by piles between PRF and PG for Low rise and High rise buildings with 2 levels of basement. The FS of piles (total pile capacity/ total design load) are also included. It is seen that the FS of high rise building is larger than that of low rise building. This may be the cause that the load shared by piles in the high rise building becomes larger. The FS of piles in the design may have strong influence on the load shared by piles in PRF and should then be further studied. To enhance the efficiency of the PRF, the design with lower FS of piles should be done. This can be done by increasing the spacing between piles in engineering practice. This topic is under investigation by the authors.



Figure 11 Load shared by piles of PRF and PG between Low rise and High rise buildings with II basements

5.4 Effect of load sharing ratio of piles in PRF on ratio of settlement of PRF against PG

Figure 12 shows the relationship between the load sharing ratio by piles in PRF and the ratio of settlements of PRF and PG (S_{PR}/S_{PG}). It illustrates the settlement behaviour of PRF in comparison to the PG for various load sharing ratios. It is seen that the relationships between the load sharing ratio and S_{PR}/S_{PG} in both cases are approximately linear functions in the range considered in this study. This implies that the settlement of PRF linearly decreases from that of the PG with decreasing load sharing ratio. This indicates that if the raft shares more load, the settlement of the PRF would decrease. It is noted that the results of high rise building case in this study cover only limited range of load sharing ratio. To validate this finding, wider range may be necessary for future study. However, with the results obtained, the slope value of this relationship of high rise building seems to be slightly steeper. This means that, with the same change of load sharing ratio, settlement reduction ratio of the high rise building case becomes less pronounced.



Figure 12 Relation of load sharing ratio by piles in PRF and the ratio of settlements of PRF and PG low rise and high rise buildings

5.5 Deformation behaviour of soil surrounding PRF and PG with various raft levels

In this section, the deformation behaviour of soil surrounding the PRF and PG in term of contour of total displacement was investigated. Comparisons on the contours of total displacement of soil surrounding the foundations of the low rise building between two considered systems (PRF and PG) for different raft levels are shown in Figures 13 and 14, respectively.



Figure 13 Contours of total displacement of soil and foundation of low-rise PRF, raft level and applied loads on raft (a) 0 m., UDL140; (b) -4m., UDL146; (c) -8m., UDL152; (d) -10m., UDL158



Figure 14 Contours of total displacement of soil and foundation of low-rise PG, raft level and applied loads on raft (a) 0 m., UDL140; (b) -4m., UDL146; (c) -8m., UDL152; (d) -10m., UDL158

The figures show the results at their design loads, which are different. It is seen that the settlement of the raft in the PRF is less than that in the PG, whereas the settlement of the soil under the raft of PRF is larger than that under the raft of PG.

This is because a portion of structural load is directly transferred to the ground under raft for the PRF while the load is only transferred along the piles for the PG.

foundations of high rise building of PRF and PG are illustrated in

The contours of total displacement of soil surrounding the

Figure 15 and 16, respectively. Like the low rise building case, the settlement of the soil under the raft of PRF is larger than that under the raft of PG. The analysis results show that even the levels of raft in PRF and

the UDLs increase, the zones of total displacement distribution are not much different for the same case. Both lateral and vertical extents of the zones are similar. However, the magnitudes of the total displacement in the central zone are much different.



Figure 15 Contours of total displacement of soil and foundation of high-rise PRF, raft level and applied loads on raft (a) 0 m., UDL350; (b) -4m., UDL356; (b) -8m., UDL362; (b) -10m., UDL368



Figure 16 Contours of total displacement of soil and foundation of high-rise PG, raft level and applied loads on raft (a) 0 m., UDL350; (b) -4m., UDL356; (b) -8m., UDL362; (b) -10m., UDL368

6. CONCLUSION

This article presents the results of numerical analyses of the PRF in the subsoil condition of north Bangkok, using 3-D FEM to investigate the effect of raft level and thickness on load shared by piles in PRF system and the settlement behaviours. Two types of buildings, low and high rise buildings with various raft levels are considered. The results analysed in terms of load shared by piles and differential settlement between pile and raft suggest the potential of using the piled raft system for low-rise building having 2 levels of basement. With this condition, the raft can carry some bearing capacity from piles around 20%. With the conditions considered in this study, for low rise building, the thickness of the raft does not have significant influence on both load sharing ratio and settlement behaviour of the overall PRF system but has influence on settlement behaviour of the raft itself. For high rise building, the thickness of the raft has influence on both settlement behaviour of the overall PRF system and the raft itself.

From further investigation on load-settlement curves, it is seen that the raft in PRF enhances not only the load capacity but also the deformation of the PRF system. However, with the assumptions set up in this study, it seems that the FS of piles has a strong influence on the effectiveness of PRF. Further study with less FS should be done, particularly for high rise building case, to confirm effectiveness of PRF. Furthermore, this study considers only the short term behaviours. The future work considering the long term condition with regard to the consolidation should be done to confirm the potential of using this system in Bangkok soft ground condition.

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